DIFFERENTIAL EQUATIONS

Synopsis :

1. An equation involving one dependent variable, one or more independent variables and the differential coefficients (derivatives) of dependent variable with respect to independent variables is called a ***differential equation***.
2. A differential equation is said to be an ***ordinary differential equation*** if it contains only one independent variable and ordinary derivatives with respect to this independent variable.
3. A differential equation is said to be a ***partial differential equation*** if it contains atleast two independent variables and partial derivatives with respect to either of these independent variables.
4. The order of the highest derivative involved in an ordinary differential equation is called the ***order*** of the differential equation.
5. The degree of the highest derivative involved in an ordinary differential equation, when the equation has been expressed in the form of a polynomial in the highest derivative by eliminating radicals and fraction powers of the derivatives is called the ***degree*** of the differential equation.
6. A relation between the variables without derivatives of a differential equation is said to be a ***solution*** or ***integral*** of the differential equation if the derivatives obtained there from, the equation is satisfied.
7. A relation $\varphi(x, y, c_1, c_2, \ldots, c_n)=0$ where $c_1, c_2, \ldots, c_n$ are n arbitrary constants is said to be the ***general solution*** or ***complete integral*** of the differential equation $F(x, y, y_1, y_2, \ldots, y_n) = 0$ if $\varphi(x, y, c_1, c_2, \ldots, c_n) = 0$ is a solution of $F(x, y, y_1, y_2, \ldots, y_n) = 0$.
8. The number of arbitrary constants in the general solution of a differential equation is equal to the order of the differential equation.
9. If $\varphi(x, y, c_1, c_2, \ldots, c_n) = 0$ is the general solution of a differential equation $F(x, y, y_1, y_2, \ldots, y_n) = 0$ then $\varphi(x, y, k_1, k_2, \ldots, k_n) = 0$ where $k_1, k_2, \ldots, k_n$ are fixed constants, is called a ***particular solution*** of the differential equation $F(x, y, y_1, y_2, \ldots, y_n) = 0$.
10. **Variables Separable :** Let the given equation be $\frac{dy}{dx} = f(x, y)$. If $f(x, y)$ is a variables separable function, i.e. $f(x, y) = f_1(x)f_2(y)$ then the equation can be written as $\frac{dy}{dx} = f_1(x)f_2(y)$ $\Rightarrow \frac{dy}{f_2(y)} = f_1(x)\,dx$. By integrating both sides, we get $\int \frac{dy}{f_2(y)} = \int f(x)\,dx + c$, where c is an arbitrary constant, which is the solution of $\frac{dy}{dx} = f(x, y)$. This method of finding the solution is known as ***variables separable.***
11. A differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is said to be a ***homogeneous differential equation*** in x, y if both $f(x, y)$, $g(x, y)$ are homogeneous functions of same degree in x, y.
12. If $a' + b = 0$ then the equation $\frac{dy}{dx} = \frac{ax + by + c}{a'x + b'y + c}$ can be solved by proper grouping of the terms.
13. A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only is called a ***linear differential equation*** of the first order in y.
14. The solution of the linear differential equation $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x only is $y \cdot e^{\int p\,dx} = \int Q \cdot e^{\int p\,dx} dx + c$.

15. The factor $e^{\int p\,dx}$ is called ***integrating factor*** (I.F) of the differential equation $\frac{dy}{dx} + Py = Q$.

16. The differential equation of the form $\frac{dx}{dy} + Px = Q$ where P and Q are some functions of y only is called a ***linear differential equation*** of the first order in x. It's solution is

$$x \cdot e^{\int p\,dy} = \int Q \cdot e^{\int p\,dy} dy + c\,.$$

17. The solution of $\frac{dy}{dx} + Py = Q$ is y.(I.F) = $\int Q$.(I.F)dx + c, where I.F = $e^{\int p\,dx}$.

18. The solution of $\frac{dx}{dy} + Px = Q$ is x.(I.F) = $\int Q$.(I.F)dy + c, where I.F = $e^{\int p\,dy}$.

19. An equation of the form $\frac{dy}{dx} + Py = Q\,y^n$, where P and Q are functions of x only, is called a ***Bernoulli's equation***